### PARVATIBAI CHOWGULE COLLEGE OF ARTS AND SCIENCE (AUTONOMOUS), MARGAO - GOA SYLLABUS FOR PROGRAMME BACHELOR OF ARTS/SCIENCE IN MATHEMATICS S.Y.B.A/B.Sc. (SEMESTER-III)

Course Title: Mathematical Analysis II

Course Code: MAT-III.C-5

Marks: 100

Credits: 4

Course Pre Requisites: - Mathematical Analysis I

Aim:- To introduce one more family of functions, Integrable functions.

Course outcome:- After completion of this course students will be able to

- C01. Study the properties of Riemann integrable functions, and the applications of the fundamental theorems of integration.
- C02. Determine various applications of the fundamental theorem of integral calculus.
- C03. Classify and evaluate improper integrals.
- C04. Integrate numerically using various methods.

#### **Content:**

1] Riemann integrals:- Tagged partition, Riemann sum, Riemann integrable functions, some simple results on integrable functions using Riemann sum. (10 lectures)

2] Darboux integrals:- Upper/lower sum, integrable function, Riemann criteria of integrable function, classes of integrable functions, (15 lectures)

3] Fundamental theorems of integration and their applications (chain rule, substitution and product rule theorems) (12 lectures)

4] Improper integration (type I, type II and type III),  $\alpha$  and  $\beta$  functions. (8 lectures)

5] Numerical integration – Quadrature Rules, Trapezoidal, Mid-point, Simpson's and Weddle's rules of integration. (15 lectures)

### **References:**

- 1. Malik S.C. and Arora Sarita. *Mathematical Analysis*, Second edition. Wiley Eastern Ltd, 1994.
- 2. Apostol Tom, Calculus Vol. I. Second Edition. Wiley Students Edition, India, 2012.
- 3. Bartle Robert G. and Sherbert Donald R. *Introduction to Real Analysis*, Third Edition. Wiley Student edition.
- 4. Narayan Shanti. Differential Calculus. S. Chand and Company Pvt. Ltd. 1988.
- 5. Goldberg Richard R., *Methods of Real Analysis*. Oxford and IBH Publishing Co. Pvt. Ltd. Indian Edition, 1970.
- 6. Bhat R.D. A Textbook of Mathematical Analysis II. Vipul Prakashan, First Edition.

### Course Title: ALGEBRA-I

Course Code: MAT-III.E-1

**Marks:** 100

Credits: 4

Course Objectives: To introduce basic algebraic structures (Groups Rings and fields).

### Learning Out comes:

- C01. Understand the definition and properties of groups.
- C02. Learn about structure preserving maps between groups and their consequences.

Understand the fundamental concepts in ring theory such as ideals, quotient rings, integral domains, and fields

#### Prerequisites: Basic Algebra

#### Content

#### 1) Groups:

Definition and Examples of Groups, Elementary Properties of Groups, finite-infinite groups, Subgroups, definition and examples, Subgroup Tests, Cyclic Groups, Cosets, Properties of Cosets, Lagrange's Theorem and consequences.

#### 2) Group Homomorphisms:

Permutation Groups, Isomorphisms definition and examples, Properties of Isomorphism., Cayley's Theorem, Automorphisms, Homomorphisms, Properties of Homomorphisms, Normal Subgroups, Factor Groups, The Isomorphism theorems.

#### 3) Rings:

Rings Definition and examples, Properties of Rings, Subrings, Integral Domains, Fields, Characteristic of a Ring, Ideals and Factor Rings, Prime Ideals and Maximal Ideals, Ring Homomorphisms, properties and examples, The Field of quotients.

#### **References:**

1) Gallian J, Contemporary Abstract Algebra, Cengage Learning

2) Fraleigh J.B., A First Course in Abstract Algebra, Pearson

3) Herstein I.N., Topics In Algebra, Wiley

### Course Title: NUMBER THEORY-I

Course Code: MAT-III.E-2

Marks: 100

Credits: 4

Course Objectives: To learn about basic concepts in number theory that will help the students.

Learning outcome: Students will be able to .

C01. Understand divisibility, congruence and their properties.

C02. Apply various results of number theory to problems.

- C03. Learn about number theoretic functions and their applications.
- C04. Understand the working of certain Diophantine equations

### 1. Divisibility:

#### (10 lectures)

(15 lectures)

(10 lectures)

(15 lectures)

(10 lectures)

Division Algorithm, Greatest Common divisor, Eucledian Algorithm, Fundamental Theorem of Arithmetic

# 2. Congruence:

Basic properties, Linear Congruence, Chinese Remainder Theorem, Quadratic Congruence.

#### 3. **Fermat's Theorem:** Fermat's a nd Wilson's Theorem

4. Number Theoretic Functions:

Sum and number of divisors, Mobius function, Mobius Inversion, greatest integer function, Euler's phi function

5. Diophantine Equations:

Linear Diophantine equations ax+by=c, the equation  $x^2 + y^2 = z^2$ , Fermat's Last Theorem.

#### **References:**

- 1. Adams & Goldstein, Introduction to Number Theory, Prentice Hall
- 2. Baker Alan, A concise introduction to the Theory of Numbers, Cambridge University Press
- 3. Burton David, Elementary Number Theory, 2012, Mc Graw Hill, 7th Edition.

# (20 Hours)

#### (20 Hours)

(20 Hours)

- 4. Niven & Zuckerman, An Introduction to the Theory of Numbers, Wiley Publications.
- 5. Telang S.G. & Nadkarni M.D, Number Theory

Course Title: Combinatorics Course Code: MAT-III.E-3 Marks: 100 Credits: 4

Course Pre Requisites: - Basic set theory, Basic Algebra, Concepts of sequences and Series.

**Aim:** - This paper is introduced to inculcate lateral thinking ability in students, to give them some basic techniques to solve combinatorial problems and make them realize that there may not be readymade solutions to all the problems.

Learning Outcome: - Students will be able to

C01. Understand basic techniques and schemes to solve different combinatorial problems

C02. Analyse a given combinatorial problem and solve it by applying one of the standard techniques learnt.

#### **Unit I: - Counting principles**

Rules of sum and product, Permutation – Combination, Distributions of distinct/ identical objects, Stirling numbers of 1<sup>st</sup> and 2<sup>nd</sup> kind Stirling's formula., Pigeon hole Principle

#### **Unit II: - Generating Functions**

Generating functions for combinations, Enumerators for permutations, distribution of distinct objects in to identical cells, Partitions of integers, relations. Exponential generating function

#### **Unit III: - Recurrence Relations**

Linear recurrence relations with constant coefficients, Characteristic equation method, Solutions by technique of generating function, recurrence relations with two indices.

#### Unit IV: - The principle of inclusion and exclusion

The general formula, derangements, Permutations with restrictions on relative positions, the rook polynomials, permutations with forbidden positions.

**Text book: -** Liu C.L., Introduction to Combinatorial Mathematics, McGraw-Hill Book Company. **References:** 

- 1. Berge C, Principles of Combinatorics, Academic Press
- 2. Brualdi R.A., Introductory Combinatorics, Pearson
- 3. Chuan-Chong Chen & Khee-Meng Koh, Principles and techniques in Combinatorics, World Scientific Publishing
- 4. Knuth, Graham, Patashnik, Concrete Mathematics: A Foundation for Computer Science, Addison Wesley
- 5. Kolman B, Discrete mathematical structures, Pearson Education
- 6. Liu C.L, Discrete mathematical structures, McGraw-Hill Book Company.
- 7. Stanley R., Enumerative Combinatorics Volume 1, Springer
- 8. Tucker Alan, Applied Combinatorics, Wiley Pvt. Ltd.

# Course Title : Numerical Methods with Python

Course Code : MAT-III.E-4

## (15Lectures)

(20Lectures)

# (15Lectures)

## (10Lectures)

Marks : 100

Credits: 4

Duration : 60 hours

Prerequisite Courses : Mathematical Analysis

Course Objectives : This course covers the basic methods for finding the finite difference, solution of simultaneous equations and the techniques of Numerical Differentiation and Numerical Integration. It also deals with solution of Algebraic and Transcendental equations.

Course Outcomes : The student will be able to

- C01. Obtain numerical solutions of algebraic and transcendental equations.
- C02. Learn about various interpolating and extrapolating methods.
- C03. Solve differentiation and integration problems numerically
- C04. Write python programs for some numerical methods.

Syllabus :

### **Module I: Error Analysis**

Approximate numbers, significant figures, rounding off numbers, Error- Absolute, relative and percentage

#### **Introduction to Python**

Installation, Basics of programming, If, elif, for and while loop, simple programs involving these loops, Lists, list comprehension.

#### **Finite Differences**

Operators-  $\Delta$ , and E (Definitions and some relations among them), finite difference tables, fundamental theorem on differences of a polynomial and examples.

### **Module II: Interpolation and Extrapolation**

Newton Gregory Forward and backward interpolation formulae (with deduction of formulae) and examples (for equal intervals)

For unequal intervals- Lagrange's Formula and Newton's divided difference formula (No proof) and examples

Python program for Newton Gregory Forward and Backward interpolation formula, Lagrange interpolation formula, Newton's Divided difference formula.

### **Module III: Numerical Integration and differentiation**

Trapezoidal rule, Simpson's 1/3<sup>rd</sup> and 3/8<sup>th</sup> rules (with proof) and problems. Weddle's rule (no proof, only problems)

Numerical differentiation and examples

Python program for Trapezoidal rule, Simpson's 1/3<sup>rd</sup> rule, 3/8<sup>th</sup> rule, Weddle's rule and numerical differentiation formula (derivative at the initial point only).

### **Module IV: Numerical Solutions of Equations**

To find a real root of an algebraic or transcendental equation using Bisection method, regular falsi method, Newton Raphson method with geometrical significance and problems and method of iteration. Gauss- Seidel method to solve simultaneous equations.

# (15 Lectures)

(15 lectures)

(15 lectures)

# (15 Lectures)

Python program for Bisection method, Regula falsi method, Newton Raphson method Gauss- Seidel method.

Curve fitting : Method of least squares- fitting a line, second degree polynomial, exponential curve and examples. Python programs for fitting a line, second degree polynomial, exponential curve.

References :

- 1. Atkinson K., An Introduction to Numerical Analysis, John Wiley & Sons
- 2. Chatterji P.N, Numerical Analysis
- 3. Comte S.D and Carl de Boor, Elementary Numerical analysis An Algorithmic approach, McGraw Hill
- 4. Hildebrand F.B, Introduction to Numerical Analysis, McGraw Hill
- 5. Sastry S.S, Introductory Methods of Numerical Analysis, Prentice Hall India
- 6. Scarborough J.B, Numerical Mathematical Analysis, Oxford and IBH Publishing Company, New Delhi.

Course Title : Differential Equations I

Course Code : MAT-III.SEC-I

Marks : 100

Credits : 4

Duration: 60 hours

Prerequisite Courses : Mathematical Analysis

Course Objectives : To introduce some methods of solving ODE of first and higher order, Applications of the same in different fields.

Course Outcomes : Students will be able to

- C01. Understand the genesis of ordinary differential equations.
- C02. Learn various techniques of getting exact solutions of solvable first order differential equations and linear differential equations of higher order.
- C03. Grasp the concept of a general solution of a linear differential equation of an arbitrary order and also learn a few methods to obtain the general solution of such equations.
- C04. Formulate mathematical models in the form of ordinary differential equations to obtain possible solutions of the day-to-day problems arising in different disciplines.

Syllabus :

#### Unit 1:-

#### (15 lectures)

Introduction. Some simple situations where we come across ODE, Geometrical Meaning of ODE, Solutions of an ODE. Picard's Existence and Uniqueness theorem.

First order ODE. Variable separable, Homogeneous, Non-Homogeneous, Exact differential equations, integrating factor, linear differential equations, Bernoulli equations.

#### Unit 2:-

#### (15 lectures)

Second order differential equations, Homogeneous and non-homogeneous differential equations, complementary function, particular integral, Wronskian, Solution space, General solution, complex solutions. Some methods of solving second order differential equations (undetermined coefficients, variation of parameters, using one solution to find other).

#### Unit 3:-

#### (15 lectures)

Linear differential equations of higher order, simple examples of non-homogenous differential equations. Some application of differential equations, LR / LCR circuits, SHM (simple-damped-forced), Equation of Catenaries, Planetary Motions – Kepler's Laws.

#### Unit 4:-

#### (15 lectures)

Some numerical methods : Euler method, modified Euler method, Runge- Kutta methods (order two and order four). Picard's method of successive approximation.

### References :

Textbook: - Simmons G.F., Differential Equations with historical Notes, Tata McGraw Hill

- 1. Boyce W.E. & DiPrima R.C., Elementary Differential Equations and Boundary Valued Problems, John Wiley Pvt Ltd.
- **2.** Braun C, Differential Equations and Their Applications: An Introduction to Applied Mathematics (Texts in Applied Mathematics), springer.
- 3. Coddington E., Theory of Ordinary Differential Equations, Tata McGraw Hill
- 4. Rainville E.D., Elementary Differential Equations, Pearson

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## Course Title: LINEAR ALGEBRA

Course Code: MAT-IV.C-6

**Marks:** 100

Credits: 4

Course Objectives: This course aims to impart emphasis on Vector spaces.

Learning outcome: Students will be able to

- C01. Understand the concepts of vector spaces, subspaces, bases, dimension and their properties.
- C02. Relate matrices and linear transformations, compute eigen values and eigen vectors of linear transformations, diagonalization.
- C03. Understand inner product spaces and the process of orthogonalization.

### Prerequisites: Basic Algebra

#### Unit 1:

#### (15 lectures)

Vector space [Definition and examples], subspaces, sum and direct sum of subspaces. Linear span, linear dependence, independence and their properties. Finite dimensional vector Space Basis, dimension of a vector

space. Dimension of sum of subspaces. Existence of complementary subspace of a finite dimensional vector space. Quotient space and its dimension.

#### Unit 2:

### (15 lectures)

Linear transformation, Kernel and Range of a Linear Transformation. Matrix representation of linear transformation, composition of linear maps, change of basis, similar matrices, Rank Nullity theorem. Unit 3: (15 lectures)

Eigen values and Eigen vectors of a linear transformation and matrices. Eigen space, Algebraic and Geometric Multiplicity of an eigenvalue. Diagonalisability of an n x n matrix over IR

#### Unit 4:

(15 lectures)

Inner products spaces. Cauchy- Schwarz inequality, Triangle inequality, orthogonal vectors, orthogonal complement, orthogonal sets and bases, Gram-Schmidt Process

### **References:**

- 1. Anton H, Elementary Linear Algebra, Wiley Pvt. Ltd.
- 2. Hoffman K. & Kunze R., Linear Algebra , PHI
- 3. Kumaresan S., Linear Algebra: A Geometric Approach, PHI
- 4. Strang G., Linear Algebra and its Applications, Cengage Learning
- 5. Vasishtha A.R., Linear Algebra, Krishna Prakashan

### Course Title: ADVANCED ANALYSIS

### Course Code: MAT-IV-E-5

**Marks:** 100

### Credits: 4

Aim:- To introduce some approximations of continuous/differentiable functions.

Course outcome :- After completion of this course students will be able to

- C01. Analyse sequence and series of functions.
- C02. Learn convergence of some special functions like exponential, logarithmic, trigonometric etc.
- C03. Represent continuous functions as polynomials

## **Content:**

1] Sequence of functions:- convergence, uniform convergence, interchange theorems. (15)

2] Series of functions:- Convergence, uniform convergence, interchange theorems. Power series and their radius of convergence, Cauchy-Hadamard theorem, Differentiation and uniqueness theore. (15)

3] Some special functions.( exponential, logarithmic and trigonometric ) (12)

4] Continuity and Gauges,  $\delta$ -fine partition, step function, inverse function theorem, Weierstrass approximation theorem (using Bernstein polynomials), Dini's theorem. (18)

## **References:**

- 1. Bartle R.G, Sherberf D.R, Introduction to Real Analysis (Third Edition), Wiley
- 2. Berberian S.K., A First Course in Real Analysis, Springer
- 3. Bhatia R., Fourier Series, Hindustan Book Agency
- 4. Goldberg R.R, Methods of Real Analysis, Oxford and lbh
- 5. Kumaresan S & Kumar A, A Basic Course in Real Analysis, CRC Press
- 6. Malik S.C. and Arora, Mathematical Analysis, New Age
- 7. Pugh C.C., Real Mathematical Analysis, Springer

- 8. Rudin W., Principles of Mathematical Analysis, Tata McGraw Hill
- 9. Shanti Narayan, P.K.Mittal, A Course of Mathematical Analysis, S Chand
- 10. Somasundaram, Mathematical Analysis
- 11. Stein E.M & Shakarchi R., Fourier Analysis: An Introduction, Princeton University Press

**Course Title:** NUMBER THEORY-II

Course Code: MAT-VI E-14

Marks: 100

Credits: 4

Course Objectives: To learn about Primitive Roots, Quadratic reciprocity and continued fractions

Learning outcome: The student will be able to

- C01. Familiarise with modular arithmetic and find primitive roots of prime and composite numbers.
- C02. Understand quadratic congruences.
- C03. Express integers as sum of squares.
- C04. Use concepts of finite and infinite continued fractions.

Prerequisites: Number Theory I

Unit 1: Primitive Roots and Indices: -

Review of congruence, Chinese remainder theorem Euler phi theorem and Euler theorem. The order of an integer modulo n, Characterization of positive integers n for which primitive roots mod-n exist, The theory of Indices.( Structure of the group of units of Z/nZ as a product of cyclic groups.)

Unit 2: The Quadratic Reciprocity Law

Euler's criterion, Legendre Symbol and its Properties, Quadratic Reciprocity, Quadratic Congruences with Composite moduli

Unit 3: Representations of numbers as sums of squares.

Fermat's two squares theorem, Lagrange's four squares theorem, and statement of Warings problem.

**Unit 4**: Continued Fractions

Finite Continued fractions, Infinite continued fractions, Dirichlet's theorem on approximation of a real number by a rational number, Liouville's theorem.

### **References:**

- 1. D. Burton, Elementary Number Theory, Seventh Edition, Mc Graw Hill
- 2. Baker, A Concise Introduction to the Theory of Numbers, Cambridge University Press
- 3. Niven, H.S. Zuckerman and H.L. Montgomery, An Introduction to the Theory of Numbers, Fifth Edition, Wiley India
- 4. W.W. Adams and L.J. Goldstein, introduction to Number Theory, Prentice Hall
- 5. S.G. Telang and M.D. Nadkarni, Number Theory
- 1. Course Title : Operations Research I
- 2. Course Code : MAT-IV. E -7
- 3. Marks : 100
- 4. Credits : 4

### (15 lectures)

(15 lectures)

(15 lectures)

# (15 lectures)

#### 5. Duration : 60 hours

- Prerequisite Courses : None 6.
- 7. Course Objectives : This course aims to teach linear programming
- 8. Course Outcomes : Students will be able to

C01. Analyse and solve linear programming models of real-life situations.

- C02. Provide graphical solutions of linear programming problems with two variables, and illustrate the concept of convex set and extreme points.
- C03. Understand the theory of the simplex method.
- C04. Know about the relationships between the primal and dual problems
- C05. Learn about the transportation, and assignment problems.

#### Syllabus :

Unit 1. Linear Programming Problem

Definition of standard form, formulation of LPP, convex set and their properties, extreme points. Graphical solution of LPP (Only two variables).

Unit 2. Simplex Method:

Theorems related to simplex method .and problems. Cases pertaining to existence of multiple solutions, unbounded and no feasible solution. Big M method and two phase Simplex method Unit 3. Duality in LPP: (10 Hours)

General Primal-Duel Pair, Formulating Dual problem, Primal-duel pair in matrix Form, Duality theorems, Duality and simplex Method.

Unit 4. Post Optimal analysis:

Change in Objective function/ constraint/activity coefficients, Structural changes. (8 Hours)

Unit 5. Transportation Problems:

Mathematical formulation, condition for existence of feasible solution, rank of transportation matrix, Initial basic feasible solution by (i) NWC method (ii) Matrix-minima and (iii) VAM, Modi's method to find an optimal solution, balanced and unbalanced transportation problems. Unit 6. Assignment Problems: (7 Hours)

Mathematical formulation, Hungarian methods to solve assignment problems, balanced & unbalanced assignments problems

### **References** :

- 1. Kanti Swarup, Gupta P.K, Man Mohan, Operations research, S Chand
- 2. Loomba, Linear Programming
- 3. Taha H, Operation Research, Pearson
- 4. Vajda, Game Theory

(5 Hours)

(20 Hours)

(10 Hours)

Course Title: Theory of Probability

Course Code: MAT-VI E-15

**Marks:** 100

Credits: 4

**Course Objectives:** To understand how mathematical tools are applied to develop tools in Statistics and learn to use those tools.

Learning outcome: The student will be able to

C01. Solve the problems related to discrete and continuous probability distribution

C02. Identify the different sampling distributions

C03. Learn about the various distribution functions like binomial, Poisson, normal etc..

Prerequisites: Statistical methods.

Unit 1 Revision of Probability theory, Set theory.

**Unit 2**: Probability as a function from super set of a non-empty set to the interval [0, 1]. Axioms of Probability. Probability distributions, Discrete and continuous distributions, Probability Mass function, density function, Distribution function, Central tendencies, Expected values, Variance, Standard Deviation, Moments, Moment generating function, characteristic function, conditional expectations and distribution, random vectors and joint probability distributions, functions of random vectors, change of variable.

(5 lectures)

### (20 lectures)

Unit 3: Uniform, binomial, Poisons, Geometric, Hyper-geometric, Normal, Chi-square, beta, gamma, Studentst, F, distributions, their distribution functions, graphs, statistic, relations.(25 lectures)

Unit 4: Modes of convergence, Weak and Strong laws of large numbers, Central limit theorem (10 lectures)

### **References:**

1. P. Billingsley, Probability and measure, 2nd edition, John Willy & sons (SEA) Pvt. Ltd. 1995

2. P.G. Hoel, S.C. Port & C.J. Stone, Introduction to Probability, Universal Book Stall, New Delhi, 1998.

3. J.S. Rosenthal, A first look at Rigorous Probability Theory, World Scientific, 2000.

4. M. Woodroofe, Probability with applications, McGraw-Hill Kogakusha Ltd. Tokyo, 1975.

**Course Title:** Differential Equations-II

Course Code: MAT-V E10

**Marks:** 100

Credits: 4

**Course Objectives:** Some more techniques of solving differential equations. Introduction to PDE. **Learning outcome:** Students will be able to

C01. Find power series solutions of differential equations

- C02. Solve ordinary differential equations using Laplace transforms.
- C03. Solve systems of first order differential equation
- C04. Understand Boundary value problems, Sturm-Liouville Theorems, linear differential equation of higher order

Prerequisites: Ordinary Differential equations.

Unit 1: Power Series Solutions of DE

Series solution of first order DE, Series solution of second order Linear DE at ordinary points Regular singular points, Gauss Hyper geometric equation, Bessel's function Legendre Polynomials (20 lectures) 

 Unit 2: Laplace Transforms

 Definition and examples, Derivatives and integrals of Laplace Transforms

 Applications to DE, Convolutions
 (15 lectures)

 Unit 3: Systems of first order DE

 Homogeneous Linear Systems with Constant coefficients
 (10 lectures)

 Unit 4: Boundary value problems, Sturm-Liouville Theorems, linear differential equation of higher order.

 (15 lectures)

#### **References:**

- G.F. Simmons, Differential Equations with Application and Historical Notes, Mc Graw Hill.
- W.E. Boyce and R.C Di Prima, Elementary Differential Equations and Boundary value Problems, Ninth Edition, Wiley India
- M. Tenenbaum and H. Pollard, Ordinary Differential Equations, Dover

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Course Title: Functions of several variables

Course Code: MAT-V C7

**Marks:** 100

Credits: 4

Course Pre Requisites: Mathematical Analysis, Co-ordinate geometry.

**Aim**: - To introduce multivariate calculus, i.e. Continuity, Differentiation and Integration of functions of several variables, applications of differentiation and integration.

Learning outcome: - Students will be able to

- C01. Progress from single variable functions to several variable functions.
- C02. Learn about partial derivatives and its applications.
- C03. Understand the concept of differentiability for several variables
- C04. Solve problems using line, double integrals.

#### Unit 1 :-

#### (15 lectures)

Introduction: - Neighbourhood of a point in  $\mathbb{R}^2$  (in  $\mathbb{R}^n$ ), Open sets in  $\mathbb{R}^2$ , Sequence in  $\mathbb{R}^2$ , limit point of a sequence, Limit of a function, Continuity of a function. Functions from  $\mathbb{R}^2$  to  $\mathbb{R}$ , Examples, and graphs in three dimensions.

#### Unit 2:-

#### (15 lectures)

Directional derivatives, Partial Derivatives, their geometrical meaning, MVT, higher order partial derivatives, Equality of mixed partial derivatives, Taylor's theorem, Applications of partial derivatives, Maxima-Minima, Saddle Point, Lagrange multiplier method,

#### Unit 3:-

Differentiability of functions of two variables, sufficient condition for Differentiability, Total derivative, chain rule, Jacobian. Implicit and Inverse function theorems.

#### (15 lectures)

#### (15 lectures)

#### Unit 4:-

#### (15 lectures)

Integration. Line integral, fundamental theorem of Calculus, Double integral on rectangles & bounded regions, change of order, change of variables, calculating volumes and surface areas of simple geometrical objects.

#### **References:**

- S.R. Ghorpade & B.V. Limaye, A Course in Multivariable Calculus and Analysis, Springer
- S.C. Malik & S. Arora, Mathematical Analysis, Fourth Edition, New Age International
- G.B. Thomas Jr. and R.L. Finney, Calculus and Analytic Geometry, Pearson
- W.H. Fleming, Functions of Several Variables, Springer
- T. Apostol, Calculus Volume II, Wiley India

Course Title: Metric Spaces

Course Code: MAT-V E-9

**Marks:** 100

Credits: 4

**Course Objectives:** To introduce different kind of 'Distance' and analysis that follows.

Learning outcome: Students will be able to

C01. Understand several standard concepts of metric spaces

- C02. Explain properties of metric spaces like openness, closedness, completeness, Bolzano-
  - Weierstrass property, compactness, and connectedness.
- C03. Learn about functions defined on metric spaces

### Prerequisites: Basic Real Analysis.

Unit1] Idea of 'Distance', definition, examples of Metric Spaces. Basic terminologies – neighbourhood of a point, open ball, open set, closed set, interior point, exterior point, limit point, isolated point, cluster point, closure of a set, interior of a set, frontier (boundary) of a set, Dense set, nowhere dense set, bounded set, diameter of a set, distance between the sets, distance of a point from a set and Results involving these concepts.

#### (15 lectures)

Unit2] Equivalence of metrics, geometry with different metrics, Subspace of a metric space, results on subspaces of metric space, sequences in a metric space, convergence of sequences, Cauchy sequence, concepts of complete metric space, completion of a metric space. (15 lectures)

Unit3] Connected subsets of a metric space, Separation of a set, connected components of metric space, Results regarding connectedness of a metric space. (10 lectures)

Unit4] Compact metric space, open cover, finite cover, sequential compactness, one point compactification, (10 lectures)

Unit5] Functions on metric space. Continuous functions (maps), equivalent definitions, invariance of compactness, completeness, connectedness of continuous images. Open maps-closed maps and their basic properties. (10 lectures)

#### **References:-**

- 1. E.T. Copson, Metric Spaces, Cambridge University Press
- 2. G.F. Simmons, Introduction to Topology and Modern Analysis, Mc Graw Hill
- 3. S. Shirali and H.L. Vasudeva, Metric Spaces, Springer
- 4. S. Kumaresan, Topology of Metric Spaces, Narosa

**Course Title:** Graph theory Course Code: MAT-V E-11 Marks: 100

# Credits: 4

Course Pre Requisites: Basic set theory, Principle of counting, Principles of Mathematical Induction Course objective: To introduce the concept of discrete graphs, trees and some of their applications in real world problems.

Course outcomes: The student will be able to

- C01. Appreciate the definition and basics of graphs along with types and their examples.
- C02. Understand the definition of a tree and use various algorithms to find the shortest path
- C03. Know the applications of graph theory to network flows.
- C04. Understand the notion of planarity and colouring of a graph.

#### Unit 1 Graphs

The Konigsberg 7 bridges problem and Euler's solution to it.

Definition of graph examples and types of graphs: complete, directed, bipartite, multi graph, etc, degree of a vertex, Adjacency matrix, incidence matrix, operations on graphs: subgraphs, union, intersection and Cartesian product, isomorphisms in graphs, special graphs: self-complimentary graphs, Petersen graphs, Herschel's Graph

#### Unit 2 Walks paths and cycles

Definition of walks, paths and cycle, distance and weighted distance, radius and diameter of a graph, eccentricity of a vertex. Euler walks, Hamiltonian cycles application: travelling salesman problem

#### Unit 3 Connectivity, cut-points, blocks, connectivity

#### Unit 4 Trees

Definition of tree, results on trees, spanning tree, application: minimum spanning trees and Kruskal's algorithm. Breadth First Search Algorithm and its use in the Chinese postman problem. Dijkstra's Algorithm for shortest path.

#### Unit 5 Coloring

Vertex coloring, chromatic number of a graph and simple relation, chromatic polynomial and its properties, edge colorings.

#### Unit 6 **Planarity**

Representations and crossing number of graphs, Euler's formula, maps and planar graphs. Proof of five color theorem for planar graphs

### **References:**

- 1. W.D. Wallis, A Beginners Guide to Graph Theory, Birkhauser
- 2. Robin J. Wilson Introduction to Graph Theory, Longman Group Ltd.
- 3. J.A. Bondy, U.S.R Murthy, Graph Theory, Springer
- 4. D.B. West, Introduction to Graph Theory, second edition, Prentice Hall of India.
- 5. F Harary, Graph Theory, Narosa
- 6. V.K. Balakrisnan, Graph Theory (Schaum series), Mc Graw Hill India

**Course Title:** Cryptography Course Code: MAT-IV E-8 Marks: 100

# (10 Lectures)

### (10 Lectures)

(12 Lectures)

(8 Lectures)

# (8 Lectures)

# (12 Lectures)

Credits: 4

Course prerequsites: Algebra-1 and Number Theory-1

Course objectives: - To introduce some applications of algebra and number theory to students.

Learning Outcome: - Students will be able to

- C01. Understand the difference between classical and modern cryptography.
- C02. Learn the fundamentals of cryptography, including Data and Advanced Encryption Standards and RSA.
- C03. Encrypt and decrypt messages using block ciphers
- C04. Know about the aspects of number theory which are relevant to cryptography.

Unit 1: Review of some topics of Number theory and algebra :-

Divisibility and primes, Prime factorization, Euclidian algorithm, Fermat's little theorem, Congruence and ring of integers.

Groups of compositions, order of permutation, Fields, finite fields, ring of polynomials, factorization of polynomials, to be reviewed.

Unit 2: Classical Cryptosystems.

Affine cryptosystem, Hill cryptosystem, Block Ciphers, Stream Ciphers, Linear feedback shift registers. Unit 3: Public Key cryptosystem. (16Lectures)

One way functions, Trapdoor functions, RSA Public Key cryptosystem, Key exchange protocols, hash functions. Unit 4: Private Key cryptosystem. (8Lectures)

Modern techniques and algorithms like DSE and AES.

Unit 5: Elliptic curve cryptosystem.

Introduction to elliptic curves and its application to factorization and cryptography.

#### **References:**

1. N. Koblitz, a course in Number theory and Cryptography, Springer.

- 2. J Katze & Y Lindell, Introduction to modern cryptography, Springer.
- 3. C Paar & J Pelze, Understanding Cryptography: A textbook for students and practitioners, Springer.
- 4. W Trappe, Introduction to Cryptography with coding theory, Pearson.

Course Title: Logic and Boolean algebra. Course Code: MAT- (Sem IV) Marks: 100 Credits: 4 Course objective: To formalize logic and set theory and introduce students to Boolean algebra. Course outcome: At the completion of the course students will be able to 1. Use symbolic logic and illustrate it. 2. Appreciate set theoretical relations and examples. 3. Apply Boolean algebra in some real-life situations. Prerequisites: Basic Algebra Content

#### Unit 1: Logic –

1.1 Revision: Statements in logic, symbolic representation, connectives, truth-tables.

15 lectures

(12Lectures)

(16Lectures)

(8Lectures)

- 1.2 The logic of compound statements: Logical form and logical equivalence, conditional statements, duality law, normal forms, rules of inference, valid and invalid arguments.
- 1.3 Quantified statements: Predicates and quantified statements, universal quantifiers, existential quantifiers, statements with multiple quantifiers, arguments with quantified statements.
- 1.4 Study of logic gates: AND, OR, NOT, XOR, XNOR, NAND and NOR gates.

#### Unit 2: Sets –

- 2.1 Revision: Basic concepts of set theory, finite and infinite sets, set operations, laws of set theory.
- 2.2 Binary relations, types of relations, equivalence relations, Partial ordering relations, posets, Hasse diagrams, upper bound, lower bound, lub, glb.

**15 lectures** 

**30 lectures** 

#### Unit 3: Lattices & Boolean algebra -

- 3.1 Lattice as a poset, duality principle for lattices, properties of lattice.
- 3.2 Sub-lattice, complemented lattice, distributive lattice,
- 3.3 Lattice homomorphisms & isomorphisms, order preserving homomorphisms.
- 3.4 Boolean algebra, its properties, sub-algebra, direct products, homomorphisms, jointirreducible elements.
- 3.5 Boolean expressions, equivalent boolean expressions, minterm, maxterm, values of boolean expressions, Stone's representation theorem for finite boolean algebra,
- 3.6 Canonical forms: sum-of-products, product-of-sums canonical forms.
- 3.7 Boolean functions and their representations- cube notation, Karnaugh maps, applications. References:
- 1. Discrete mathematical structures with applications to computer science by J.P. Tremblay, R. Manohar (Principle text)
- 2. Elements of Discrete mathematics by C. Liu
- 3. Discrete mathematics (Schaum's outlines) by Seymour Lipschutz, Marc Laras Lipson & Varsha H. Patil.
- 1. Course Title : Operations Research II
- 2. Course Code : MAT-V. E -13
- 3. Marks : 100
- 4. Credits : 4
- 5. Duration : 60 hours
- 6. Prerequisite Courses : Operations Research I
- 7. Course Objectives : This course aims to teach more methods of OR.
- 8. Course Outcomes : Students will be able to

- C01. Solve game theory using different methods
- C02. Evaluate EOQ in different inventory models.
- C03. Evaluate various measures of performance in queueing models
- C04. Find better approximations of results using simulations

Syllabus :

Unit 1. Game Theory: (15 Hours) Optimal Solution of Two-Person Zero-Sum Games, Solution of Mixed Strategy Games, Graphical solution of 2 x n and 2 x m Games, arithmetic method for n x n games, general solution of m x n games, Converting Game theory into LPP. Unit 2. Inventory Control: (15 Hours) Types, Reasons, Objective and the Factors affecting inventory control, Concept of EOQ, deterministic Inventory problem with/without shortage, Price Breaks, Multi-item deterministic problem. Uncertain demands, one period problem with / without set-up cost. Unit 3. Queueing Theory: (15 Hours) Elements of Queueing system, Probability Distribution in queuing system, Classification of queuing system, queuing models, Transient and Steady states, Poisson/ non-Poison queuing systems, Cost model in queuing. Unit 4.Simulation: (15 Hours) Need of simulation, prosses of simulation, simulation models, Event type of simulation, generation of random numbers, Monte-Carlo simulation, Simulation of – Inventory/ Queuing/ Maintenance problems. Simulation in investments, budgeting and job sequencing.

References :

- 1. Kanti Swarup, Gupta P.K, Man Mohan, Operations research, S Chand
- 2. Loomba, Linear Programming
- 3. Taha H, Operation Research, Pearson
- 4. Vajda, Game Theory

#### PARVATIBAI CHOWGULE COLLEGE OF ARTS AND SCIENCE (AUTONOMOUS), MARGAO - GOA SYLLABUS FOR PROGRAMME BACHELOR OF ARTS/SCIENCE IN MATHEMATICS T.Y.B.A/B.Sc. (SEMESTER-VI)

Course Title: Vector Analysis

Course Code: MAT-VI C-8

**Marks:** 100

Credits: 4

**Course Objectives:** To introduce students to Vector representations of geometrical objects, analysis done on them and their applications in Physics.

Learning outcome: Students will be able to

- C01. Define Vector valued functions and various concepts like gradient, divergence and curl of these functions
- C02. Analyse vector functions to find limits, derivatives, tangent lines, integrals, arc length, curvature, torsion
- C03. Evaluate line integrals, surface area, surface integrals and volume integrals
- C04. Apply Green, Gauss and Stokes' theorems to solve problems

Prerequisites: functions of several variables.

Unit 1: Revision of vectors :- Basic concept such as dot product, cross product, scalar triple product, vector triple product, geometrical understanding of all algebraic operations, orthogonal vectors, vector equations of geometrical objects, (10 lectures)

Unit 2: Vector valued functions (vector fields), scalar valued functions (scalar fields), concepts of 'curves', 'plane', 'surface' in  $\mathbb{R}^3$  Idea of continuous, smooth and regular objects in  $\mathbb{R}^3$ , Gradient, Divergence and Curl of these functions, Physical interpretations. Irrotational and solenoidal vector fields.

#### (20 lectures)

**Unit 3:** Line integral, surface integral and volume integral, (arc length, surface area, and volume of simple objects), Green's theorem, Stokes theorem, Gauss Theorem and Green's formulas.

#### (20 lectures)

Unit 4: Theory of Curves in R³, Unit speed curve, Tangent, Normal, Bi-normal, Curvature, Torsion, Ferret-<br/>Serrate formulae, evolutes and involutes.(10 lectures)Defense oneDefense one

**References:** 

1 .H.F. Davis and A.D. Snider, Introduction to Vector Analysis, Sixth Edition, Allyn & Bacon

2. J.E. Marsden and A. Tromba, Vector Calculus, Sixth Edition, W.H. Freeman Publishers

3. E. Kreysig, Advanced Engineering Mathematics, Tenth Edition, Wiley Inida

4. M. Spiegel, Vector Analysis: Schaum's Outline Series, Mc Graw Hill

**Course Title:** COMPLEX ANALYSIS

Course Code: MAT-VI E-13

Marks: 100

Credits: 4

**Course Objectives:** Introduce students to complex numbers, complex functions and complex differentiation and integration.

Learning outcome: The student will be able to

- C01. Understand complex numbers and their representations
- C02. Understand the significance of differentiability and analyticity of complex functions
- C03. Evaluate contour integrals.
- C04. Compute Taylor and Laurent series expansions of analytic functions,
- C05. Classify the nature of singularity, poles and residues and application of Cauchy Residue theorem.

Prerequisites: Basic Real Analysis, Mathematical Analysis, Functions of several Variables

#### **Unit 1: Complex Numbers**

Algebraic properties of complex numbers, modulus, Argand diagram, exponential form and polar coordinates, triangle inequality and metric properties, connectedness of regions. (10 lectures)

#### **Unit 2: Analytic Functions**

Complex valued functions on complex domain, limits and continuity of complex valued function on a complex domain, differentiability and analytic functions, algebra of analytic functions, Cauchy-Riemann equations, sufficient condition for analyticity, Harmonic Functions. (10 lectures)

### **Unit 3: Elementary Functions**

Exponential, logarithmic function and its branches, trigonometric functions, hyperbolic functions, complex exponents and roots. (10 lectures)

#### **Unit 4: Contour Integration**

Contours and contour integrals, Cauchy Goursat's theorem, Simply connected domains, Cauchy's integral formula, higher derivatives of analytic functions, Liouville's theorem, fundamental theorem of algebra, maximum modulus principle. (10 lectures)

#### Unit 5: Series

Convergence of series, Taylor series, Laurent series.

### (10 lectures)

# Unit 6: Residue Theory

Singularities of a function, poles and essential singularities, residues at a singular point and its computation, Cauchy residue theorem (10 lectures)

## **References:**

- 1. R.V. Churchill and J.W. Brown, Complex variables and Applications, Eight Edition, Mc Graw Hill
- 2. L. V. Ahlfors, Complex Analysis, Mc Graw Hill
- 3. A.R. Shastri, Complex Analysis, Laxmi Publications
- 4. M. Spiegel and S. Lipschutz, Complex Variable: Schaum's Outline Series, Mc Graw Hill
- 5. J.B. Conway, Functions of a Complex Variable, Narosa
- 6. S. Ponnusamy, Complex Analysis, Narosa
- 7. S. Ponnusamy and H. Silverman , Complex variables with Applications, Birkhauser
- 8. T.W. Gamelin, Complex Analysis, Springer
- 9. E.M. Stein and R. Shakarchi, Complex Analysis, Princeton Lectures in Analysis

Course Title: Algebra II Course Code: MAT- (Sem 6) Marks: 100 Credits: 4

Course Objectives: - To further the knowledge of algebraic structures.

Learning Out comes: the students will be able to

C01. Recognize and use the Sylow theorems to characterize certain finite groups.

C02. Learn in detail about polynomial rings,

C03. Understand fundamental properties of finite field extensions, and classification of finite fields

Prerequisites: Basic Algebra, Algebra I

#### Content

#### 1) Sylow Theorems

Orbit- Stabiliser theorem, Conjugacy classes, The Class equations, The Sylow Theorems and their Applications 3)Polynomial Rings: (25 Hours)

Definition, The Division Algorithm and its consequences, factorisation of Polynomials, reducibility and irreducibility tests, Eisenstein criteria, Unique factorization in Z[x], Irreducible element, Prime element, UFD, PID, ED.

#### 4) Finite Fields

(10 Hours)

(25 Hours)

Definition, examples, Classification of finite Fields,  $Z_p$ ,  $Z_p^n$ , Special case study when p=2.

### **References:**

1) Gallian J, Contemporary Abstract Algebra, Cengage Learning

2) Fraleigh J.B., A First Course in Abstract Algebra, Pearson

3) Herstein I.N., Topics In Algebra, Wiley

**Course Title: COMPUTATIONAL LINEAR ALGEBRA** 

Course Code: MAT-IV E-17

Max Marks: 100

Credits: 4

Course Objectives: To learn about the different computational techniques in linear algebra

Learning Outcome: The student will be able to

C01. Use Gaussian Elimination to understand the system of linear equations

C02. Understand the orthogonal vectors

C03. Test a matrix for positive definiteness

C04. Find the singular value decomposition of the matrix

C05. Understand matrix norm and condition number

Unit 1: Review of Gaussian Elimination	(8 lectures)
Unit 2: Orthogonality	(10 lectures)
Orthogonal vectors and subspaces, projections and least squares, QR Factorizations	
Unit 3: Positive Definite Matrices	(20 lectures)
Maxima, minima and saddle points, test for positive definiteness, singular value decompositions, Lagrange	
multipliers and Rayleigh quotients, finite element method	
Unit 4: Matrix norm and Condition Number	(22 lectures)
References:	
1 G Strang Linear Algebra and its Applications	

1. G. Strang, Linear Algebra and its Applications

Course Title : Computers for Mathematics

Course Code : MAT-III.E-16

Marks : 100

Credits : 4

Duration : 60 hours

Prerequisite Courses : Basic knowledge of computers, a basic course in ODE, Numerical analysis. Familiarity with computer programming will be helpful but not compulsory.

Course Objectives : To train students to use computers for mathematical typing, computing and plotting. Helping them understand theory using computational methods

Course Outcomes : Students will be able to

- C01. Type mathematics using Latex
- C02. Understand basic aspects of programming.
- C03. Use programming languages to solve mathematical problems Perform basic data analysis using R

Syllabus :

#### **Unit 1.Introduction to LaTeX**

Mathematical typing in MS Word 07/13 Short comings of Word, need of some other package of writing. Document Editing using LaTeX: - Text formatting, Paragraph formatting Fonts and colours, special char acters, Tables, Page layout, importing graphics, footnotes, referencing, Beamer Presentations (10 Lectures)

### Unit 2.Mathematical typesetting in LaTeX

Mathematical environments and packages Symbols, Greek letters and operators, Powers and indices, fractions, Roots, Sums, integrals and derivatives, Brackets, Matrices

### Unit 3. Introduction R Programming.

Fundamentals of R programming:-Introduction to interface (R studio), evolution, features, All Data types in R, Variable (assignment, its data type), All operators in R, Loops and R functions (user defined and built in functions), Entering data from keyboard, Importing data from various data Sources. (12 Lectures)

Unit 4. R Statistics, data visualization and management

R charts and graphs, Data frame, Data reshaping, Mean, Median, Mode.

### Unit 5. Scilab/ Matlab/ Geogebra

Introduction to Scilab, familiarizing with the Scilab, Scilab syntax, Variables, functions, Loops, plotting graphs.

References :

1. M.Alley, the Craft of Scientific Presentations, Springer (2003).

(11 Lectures)

(15 Lectures)

#### (12 Lectures)

- 2. W.C. Booth, G.G. Colomb, and J.M. Williams, The Craft of Research (Chicago Guides to Writing, Editing, and Publishing) Univ. of Chicago Press, 2008.
- 3. George Graetzer, Math into LaTex, An introduction to LaTex and AMS-LaTex, Birkhauser, 1996.
- 4. Donald E. Knuth; Tracy E Larrabee, Paul M. Roberts: Mathematical writing, Mathematical Association of America, 1989.
- 5. S. Krantz: A Primer of Mathematical Writing: Being a Disquisition on Having Your Ideas Recorded, Typeset, Published, Read & Appreciated, American Mathematical Society, 1996.
- 6. S. Krantz: How to Teach Mathematics, American Mathematical Society, 1999.
- 7. Leslie Lamport: LaTex, a document preparing system, Addison-Wesley, 1994.
- 8. Jr. Strunk, William; E. B. White, The Elements of Style, Fourth Edition, Longman; 4th edition (1999).
- 9. Robert I. Kabacoff(2011): R in Action Data Analysis and graphics in R, Manning.
- 10. Hadley Wickham(2009):ggplot: Elegant graphics for Data Analysis, Springer.

### **Course Title:** PEDAGOGY OF MATHEMATICS

Course Code: MAT-V E-12

**Marks:** 100

Credits: 4

**Course Objectives:** To make the students aware of the different methods used for the teaching of mathematics. **Learning outcome:** Students will be able to

(10 lectures)

- C01. Understand the nature and value of mathematics
- C02. Learn different methods and techniques of teaching mathematics
- C03. Find the appropriate method to teach various topics of school mathematics

### Unit 1: Meaning and History of Mathematics

Meaning of Mathematics **Branches of Mathematics** Historical Development of Mathematics **Unit 2: Methodology of Teaching Methods** (40 lectures) Inductive - Deductive Analytic-Synthetic Heuristic/ Discovery Project Method Open Ended Approach Investigative Approach **Problem Solving** Concept Attainment Model (With applications in different topics in School level Mathematics syllabus) **Unit 3: Techniques of Teaching** (8 lectures) Assignments Drill work

Remedial teaching Accelerated teaching **Unit 4: Qualities of a good Mathematics Teacher** 

#### **References:**

- 1. E.T Bell, Men of Mathematics, Touchstone
- 2. M. Ediger, Essays on teaching mathematics, Discovery Publishing Pvt.Ltd
- 3. R.G. Goel, Teaching of Mathematics, Lotus Press
- 4. A. James., Methods of teaching Mathematics, Neelkamal
- 5. Kline Morris, Mathematical Thought From Ancient to Modern Times, Oxford University Press
- 6. S. Krantz, How to teach Mathematics, American Mathematical Society
- 7. G. Polya, How to Solve It, Penguin UK
- 8. Sidhu Kulbir Singh., Teaching of Mathematics, Sterling Publishers Pvt. Ltd
- 9. J. Stillwell, Mathematics and its History, Springer
- 10. P.P Zubair., Teaching of Mathematics, Aph Publishing

(2 lectures)