Parvatibai Chowgule College of Arts & Science,
Gogol, Margao, Goa

Department of Mathematics

Course Structure and Syllabus

Course Structure for Mathematics Major

	Core	**Core**					
Sem-I	Basic Algebra	Basic Real Analysis	---	---	---	---	
Sem-II	Coordinate Geometry	Mathematical Analysis	---	---	---	---	
			Elective-I	**Elective-II**	**Elective-III**	**Elective-IV**	**Elective-IV**
Sem-III		Differential Equations- I	Abstract Algebra-I	Number Theory-I	Combinatorics	Numerical Methods	
Sem-IV		Linear Algebra	Advanced Analysis	Abstract Algebra-II	Operations Research	Cryptography	
Sem-V		Functions of Several Variables	Metric Spaces	Differential Equations-II	Graph Theory	Pedagogy of Mathematics	
Sem-VI		Vector Analysis	Complex Analysis	Number Theory-II	Probability Theory	Computers for Mathematics	Computational Linear Algebra

Course Structure for Mathematics Minor

Semester	**Core (Minor)**
I	Basic Algebra
II	Coordinate Geometry
III	Real Analysis/ Differential Equations –I
IV	Mathematical Analysis/ Linear Algebra
V	Graph Theory / Numerical Methods
VI	Operations Research/ Probability Theory/ Vector Calculus

SYLLABUS

Course Title: BASIC ALGEBRA

Course Code: MAT-I.C-1

Marks: 100

Credits: 4

Course Objectives: Introduction to the basic concepts of Algebra which are used in mathematics.

Learning outcome: Student will be well equipped with all the concepts which are useful to understand mathematics

1) Sets (3 lectures)

Notion of Sets- Subsets - Power sets - Equality of sets - Finite and Infinite sets - Set operations - De-Morgan's laws - Cartesian product of sets

2) Relations and Functions (9 lectures)

Relations - Types of relations - Binary relation - Equivalence relation - Equivalence classes and partitions- Congruence modulo n – Mappings - One-One and onto mappings - Composition of Mappings - Identity and Inverse mappings - Binary Operations in a set.

3) Logic (3 lectures)

Logical statements - connectives - truth tables - tautologies

4) Matrices and determinants (25 lectures)

Equality of matrices- Addition of Matrices- Multiplication of Matrices- Properties of multiplication- Transpose of matrix- Conjugate of a matrix- Determinant of a square matrix- Singular and non-singular matrices- Symmetric and skew symmetric matrices- Hermitian and skew Hermitian matrices- minor and cofactor of an element of a matrix- Ad joint of a square matrix- Inverse of a square matrix- orthogonal and unitary matrices- Elementary Row, Column operations- Elementary matrices- inverse of a matrix using elementary operations- Gaussian Elimination- Linear Equation: system of homogeneous equations- Consistency and solution of a system of linear equations- -inverse of a square matrix using Gaussian Elimination- Rank of a Matrix- Normal Form- Row-Echelon matrix- Row rank and column rank of a matrix

5) Equations (20 lectures)

Algebraic equations- general properties- Fundamental theorem of Algebra(statement only)- Nature of roots of an equation (surd or complex roots occur in pairs)- Statement of Descartes' rule of signs and applications- relations between roots and co-efficients- transformation of

equations- reciprocal equations- algebraic solution of cubic equation-Cardan's method- Multiplicity of roots.

References:

1. R.D. Bhatt, Algebraic Structures, Vipul Prakashan
2. C.L.Liu, Discrete Mathematical Structures,
3. Shanti Narayan and P.K.Mittal, A textbook of Matrices, S. Chand and Company
4. H.S. Hall and S.R.Knight, Higher Algebra, AITBS Publishers
5. K.B.Datta, Matrix and Linear Algebra, PHI
6. M. Artin, Algebra, PHI
7. S. Lang, Introduction to Linear Algebra, Second Ed., Springer-Verlag

CourseTitle: BASIC REAL ANALYSIS

Course Code: MAT-I.C-2

Marks: 100

Credits: 4

Aim: - Introduction to Real numbers and real valued functions of a Real variable.

Learning Outcome: - Students will be well versed with all properties of real numbers, geometry of real line, notations, and functions.

Methodology: -Lecturing. Standard examples to make ideas clear, proving results for self-study.

Course Duration: 60 Lectures

Unit 1: Preliminaries: (4 Lectures)

Prepare students with the required background for real analysis

a. Set theory: Sets, operations on sets, sets of number systems
b. Functions: Maps and relations, Function, Inverse, Compositions, restriction and extensions of functions, How to graph a function
c. Counting: - Principle of Mathematical induction. Finite & infinite sets, countable sets and their properties.

Unit 2: The Real number system R: Introduce the real number system. (12 Lectures).

a. Algebraic Properties, Order Properties
b. Inequalities (Triangle and associated inequalities

c. Neighborhood, Intervals and their properties

d. What is an ε-neighborhood

e. Supremum and Infimum, Bounded functions and their properties, ArchimedeanProperty, Density Property, R is uncountable

Unit 3: Limits: Introduce the concept of limit used in analysis. (8Lectures)

d. Concept of limit point and cluster point

e. Limit theorems

f. One sided limits, Infinite limits

2) **Sequences of Reals**: To introduce Real sequences (14Lectures)

a. Definitions and limit: :- Definition, Limit of a sequence, What do you mean by convergence of sequence, Tail of a sequence

b. Algebra of Sequences :- Algebra of limits, Inequalities, Sandwich theorem

c. Monotone Sequences Monotone convergence theorem

d. Subsequences, Bolzano Weirstrass Theorem for sequences

e. Cauchy Sequences: - What is a Cauchy Sequence, Properties, Cauchy General Principle for convergence of sequences, R is complete.

f. Sequential convergence criterion for limit of functions

3) **Continuous Functions**: To study the behaviour of continuous functions (12Lectures)

a. Definition and concept of continuous functions, Sequential criterion for continuity of functions, Algebra of continuous functions, Behaviour on Intervals, Concept of uniform continuity

b. Graphs of Some continuous functions (Monotone, Inverse, Power/Root function)

4) **Series**: Introduce Series of Reals (6 Lectures)

- Definition, Properties, Series with non-negative terms, Test for convergence (n^{th} term test, Comparison test Ratio test, Root test).

References:

1) ,R.G.Bartle and D. Sherbert, Introduction to Real Analysis, Wiley

2) Robert Sticartz, The Way of Analysis, Jones and Bartlett Publishers

3) T. Apostol, Calculus (volume I), Wiley Eastern Ltd.

4) S.C. Malik, Savita Arora, Mathematical Analysis, New Age International Publishers

5) J.R. Munkres, Topology,Prentice Hall of India

Course Title: Coordinate Geometry

Course code: MAT-II C3

Marks: 100

Credits: 4

Aim: To learn the different 2 and 3 dimensional geometry, tracing of curves and space curve

Learning Outcome: the student will be able to identify and trace the various conic sections, trace curves in Cartesian and polar coordinates

1. Co-ordinate systems: -Cartesian, Polar, Cylindrical and Spherical coordinates systems, Relations between them. **(3 lectures)**

2. Concepts of tangent, normal, line, Plane, Sphere, Cone and Cylinder. Equation of all and forming equation under desired conditions, relationship between them. **(10 lectures)**

3. Central conics: -ellipse, parabola and hyperbola. Their equations, properties and their graphs. **(12 lectures)**

4. General equation of second degree: - General conics, conic sections their equations, properties and their graphs. **(15 lectures)**

5. Change of origin, rotation of axis and translation of axis and its effect on equation of geometrical object. **(5 lectures)**

6. Tracing of curves using concepts of derivatives, concavity-convexity, singular points (double point, cusp, and node), and monotonicity of function, polar co-ordinates, and asymptotes.

(15 lectures)

References: -

• P.K.Jain, Khalil Ahmad- Analytic Geometry of three dimension- 2nd edition –Wiley Eastern Ltd. (1991)

• Loney S.L. - The Elements of Coordinate Geometry (Part I): CartesianCoordinates – MacMillan

• Shanti Narayan, P.K.Mittal – Differential Calculus – S. Chand Publications.

• Gibson C.G. Elementary Euclidean Geometry: An undergraduate introduction- Cambridge University Press.

Course Title: MATHEMATICAL ANALYSIS

Course Code: MAT-II.C 4

Marks: 100

Credits: 4

Course Objective: To Study

The differentiable functions, their properties and some applications.

The idea of Riemann integration and some rules of integration.

Learning Outcomes:

To make the students comfortable with the differentiations and integrations that are essential in almost all the branches and also can make use of these ideas for Geometry

I. Derivatives of Functions (8 lectures)

- Definitions

Derivative at a point, Differentiability in an interval, Derivative of a function, meaning of the sign of derivative, Geometrical meaning of the derivative and higher order derivatives.

- A necessary condition for the existence of a finite derivative (theory and problems)

II. Uses Properties of differentiable functions: (14 lectures)

- Algebra of derivatives
- Derivative of the inverse function
- Darboux property, Darboux theorem and intermediate value theorem.
- Rolle's Theorem, Lagrange's Mean value theorem, Cauchy's MVT and

Their applications, Taylor's and Maclaurin's theorem.

- Increasing and decreasing functions.

III. Applications (14 Lectures)

- Approximations
- Extreme values of a function: investigation of the points of maximum and minimum values.
- L' hospital's rule (Indeterminate form, 0/0 form, 00/00 forms etc.) and some problems.

IV. Riemann Integration: (24 lectures)

4.1 Some basic terminology and notations:

Partition of an interval, upper and lower sums of bounded real valued function over I, Refinement of a partition.

4.2 Properties of partitions

4.3 Upper and lower integrals, Riemann integrals functions and some problems.

4.4. Riemann criterion for integrability and some applications based on it.

- Properties of Riemann integrable functions.

- Indefinite integral, fundamental theorems of integral calculus.
- Improper Integrals (Type I, Type II and Type III)

References:

1. Malik S.C. and Arora Sarita. *Mathematical Analysis*, Second edition. Wiley Eastern Ltd, 1994.
2. Apostol Tom, *Calculus Vol. I*. Second Edition. Wiley Students Edition, India, 2012.
3. Bartle Robert G. and Sherbert Donald R. *Introduction to Real Analysis*, Third Edition. Wiley Student edition.
4. Narayan Shanti. *Differential Calculus*. S. Chand and Company Pvt. Ltd. 1988.
5. Goldberg Richard R., *Methods of Real Analysis*. Oxford and IBH Publishing Co. Pvt. Ltd. Indian Edition, 1970.
6. Bhat R.D. *A Textbook of Mathematical Analysis II*. Vipul Prakashan, First Edition.

Course Title: Differential Equations I

Course Code: MAT-IV.C-5

Marks: 100

Credits: 4

Course Pre Requisites: - Mathematical Analysis

Aim: - To introduce some methods of solving ODE of first and higher order, Applications of the same in different fields.

Learning outcome: - Students can solve ODE with constant coefficients, given a simple situation can make an ODE.

Unit 1:- **(8 lectures)**

Introduction. Some simple situations where we come across ODE, Geometrical Meaning of ODE, Solutions of an ODE. Picard's Existence and Uniqueness theorem.

Unit 2:- **(15 lectures)**

First order ODE. Variable separable, Homogeneous, Non- Homogeneous, Exact differential equations, integrating factor, linear differential equations, Bernoulli equations.

Unit 3:- **(17 lectures)**

Second order differential equations, Homogeneous and non-homogeneous differential equations, complementary function, particular integral, Wronskian, Solution space, General solution, complex solutions. Some methods of solving second order differential equations (undetermined coefficients, variation of parameters, using one solution to find other).

Unit 4:- **(8 lectures)**

Linear differential equations of higher order, simple examples of non-homogenous differential equations.

Unit 5 :- **(12 lectures)**

Some application of differential equations, LR / LCR circuits, SHM (simple-damped-forced), Equation of Catenaries, Planetary Motions – Kepler's Laws.

Textbook: - Simmons G.F., Differential Equations with historical Notes, Tata McGraw Hill

References

1. Boyce W.E. & DiPrima R.C., Elementary Differential Equations and Boundary Valued Problems, John Wiley Pvt Ltd.
2. Braun C, Differential Equations and Their Applications: An Introduction to Applied Mathematics (Texts in Applied Mathematics), springer.
3. Coddington E., Theory of Ordinary Differential Equations, Tata McGraw Hill
4. Rainville E.D., Elementary Differential Equations, Pearson

Course Title: ALGEBRA-I

Course Code: MAT-III.E-1

Marks: 100

Credits: 4

Course Objectives: This course aims to impart emphasis on concepts of groups and rings

Learning outcome: Students will be familiar with groups, rings and their characteristics.

Prerequisites: Basic Algebra

1. **Groups** **(30 Lectures)**

Definition of a group, with examples and simple properties-Groups of Matrices- Direct Product of Groups-Subgroups—Order of a Group-Order of an element of a group- Cyclic groups-Coset decomposition-Lagrange's theorem and its consequences-Normal subgroup and

Quotient groups. Permutation groups-Cycles and transpositions-Even and odd permutations-Alternating groups

2. **Homomorphisms** **(10 lectures)**

   Group Homomorphism-Isomorphism- kernel of a homomorphism-The homomorphism theorems-The isomorphism theorems - Cayley's theorem

3. **Rings** **(20 lectures)**

   Rings and their elementary properties-Integral domain-Field-Field of quotients-Characteristic of a ring-Subrings-Ideals and their properties-Quotient Rings-Homomorphism of Rings-Prime Ideal and Maximal Ideal- Rings of Polynomials.

References:

1. Fraleigh J.B., A First Course in Abstract Algebra
2. Gallian J, Contemporary Abstract Algebra, Narosa, New Delhi
3. Gopalkrishnan N.S., University Algebra
4. Herstein I.N , Topics in Algebra,(2012) John Wiley & Sons., 2ndEdition

Course Title: NUMBER THEORY-I

Course Code: MAT-III.E-2

Marks: 100

Credits: 4

Course Objectives: To learn about basic concepts in number theory that will help the students.

Learning outcome: Students will be able to understand congruence and number theoretic functions.

1. **Divisibility:** **(10 lectures)**

   Division Algorithm, Greatest Common divisor, Eucledian Algorithm, Fundamental Theorem of Arithmetic

2. **Congruence:** **(15 lectures)**

   Basic properties, Linear Congruence, Chinese Remainder Theorem, Quadratic Congruence.

3. **Fermat's Theorem:** **(10 lectures)**

   Fermat's and Wilson's Theorem

4. **Number Theoretic Functions:** **(15 lectures)**

Sum and number of divisors, Mobius function, Mobius Inversion, greatest integer function, Euler's phi function

5. **Diophantine Equations: (10 lectures)**

Linear Diophantine equations ax+by=c, the equation$x^2 + y^2 = z^2$, Fermat's Last Theorem.

References:

1. Adams & Goldstein, Introduction to Number Theory, Prentice Hall
2. Baker Alan, A concise introduction to the Theory of Numbers, Cambridge University Press
3. Burton David, Elementary Number Theory, 2012, Mc Graw Hill, 7th Edition.
4. Niven & Zuckerman, An Introduction to the Theory of Numbers, Wiley Publications.
5. Telang S.G. & Nadkarni M.D, Number Theory

Course Title: Combinatorics

Course Code: MAT-III.E-3

Marks: 100

Credits: 4

Course Pre Requisites: - Basic set theory, Basic Algebra, Concepts of sequences and Series.

Aim: - This paper is introduced to inculcate lateral thinking ability in students, to give them some basic techniques to solve combinatorial problems and make them realize that there may not be readymade solutions to all the problems.

Learning Outcome: - Students will be able to tackle some combinatorial problems and also can analyse given problem to find a way to solve it.

Unit I: - Counting principles (20Lectures)

Rules of sum and product, Permutation – Combination, Distributions of distinct/ identical objects, Stirling numbers of 1st and 2nd kind Stirling's formula., Pigeon hole Principle

Unit II: - Generating Functions (15Lectures)

Generating functions for combinations, Enumerators for permutations, distribution of distinct objects in to identical cells, Partitions of integers, relations. Exponential generating function

Unit III: - Recurrence Relations (15Lectures)

Linear recurrence relations with constant coefficients, Characteristic equation method, Solutions by technique of generating function, recurrence relations with two indices.

Unit IV: - The principle of inclusion and exclusion (10Lectures)

The general formula, derangements, Permutations with restrictions on relative positions, the rook polynomials, permutations with forbidden positions.

Text book: - Liu C.L., Introduction to Combinatorial Mathematics, McGraw-Hill Book Company.

References:

1. Berge C, Principles of Combinatorics, Academic Press
2. Brualdi R.A., Introductory Combinatorics, Pearson
3. Chuan-Chong Chen & Khee-Meng Koh, Principles and techniques in Combinatorics, World Scientific Publishing
4. Knuth, Graham, Patashnik, Concrete Mathematics: A Foundation for Computer Science, Addison Wesley
5. Kolman B, Discrete mathematical structures, Pearson Education
6. Liu C.L, Discrete mathematical structures, McGraw-Hill Book Company.
7. Stanley R., Enumerative Combinatorics Volume 1, Springer
8. Tucker Alan, Applied Combinatorics, Wiley Pvt. Ltd.

Course Title: NUMERICAL METHODS

Course Code: MAT-III.E-4

Marks: 100

Credits: 4

Course Objectives: This course covers the basic methods for finding the finite difference, solution of simultaneous equations and the techniques of Numerical Differentiation and Numerical Integration. It also deals with solution of Algebraic and Transcendental equations.

Learning outcome: The student will be able to solve numerically various equations.

Module I: Error Analysis (7 lectures)

Approximate numbers, significant figures, rounding off numbers, Error- Absolute, relative and percentage

Module II: Finite Differences (7 lectures)

Operators- Δ, ∇ and E (Definitions and some relations among them), finite difference tables, fundamental theorem on differences of a polynomial and examples.

Module III: Interpolation and Extrapolation (7 lectures)

Newton Gregory Forward and backward interpolation formulae (with deduction of formulae) and examples (for equal intervals)

For unequal intervals- Lagrange's Formula and Newton's divided difference formula (No proof) and examples

Module IV: Numerical Integration and differentiation (11 lectures)

Trapezoidal rule, Simpson's $1/3^{rd}$ and $3/8^{th}$ rules (with proof) and problems. Weddle's rule (no proof, only problems)

Numerical differentiation and examples

Module V: Numerical Solutions of Equations (7 lectures)

To find a real root of an algebraic or transcendental equation using Bisection method, regular falsi method, Newton Raphson method with geometrical significance and problems and method of iteration

Module VI: Curve fitting (7 lectures)

Method of least squares- fitting a line, second degree polynomial, exponential curve and examples

Module VII: Solution of System of Equations (7 lectures)

Gauss Elimination, Gauss- Seidal Iteration method, Iteration method

Module VIII: Solution of Differential Equations (7 lectures)

Euler's Method, Runge Kutta Method

References:

1. Atkinson K., An Introduction to Numerical Analysis, John Wiley & Sons
2. Chatterji P.N, Numerical Analysis
3. Comte S.D and Carl de Boor, Elementary Numerical analysis - An Algorithmic approach, McGraw Hill
4. Hildebrand F.B, Introduction to Numerical Analysis, McGraw Hill
5. Sastry S.S, Introductory Methods of Numerical Analysis, Prentice Hall India
6. Scarborough J.B, Numerical Mathematical Analysis, Oxford and IBH Publishing Company, New Delhi.

Course Title: LINEAR ALGEBRA

Course Code: MAT-IV.C-6

Marks: 100

Credits: 4

Course Objectives: This course aims to impart emphasis on Vector spaces.

Learning outcome: Students will be familiar with Vector Spaces, linear transformations and inner product spaces

Prerequisites: Basic Algebra

Unit 1: (15 lectures)

Vector space [Definition and examples], subspaces, sum and direct sum of subspaces. Linear span, linear dependence, independence and their properties. Finite dimensional vector Space Basis, dimension of a vector space. Dimension of sum of subspaces. Existence of complementary subspace of a finite dimensional vector space. Quotient space and its dimension.

Unit 2: (15 lectures)

Linear transformation, Kernel and Range of a Linear Transformation. Matrix representation of linear transformation, composition of linear maps, change of basis, similar matrices, Rank Nullity theorem.

Unit 3: (15 lectures)

Eigen values and Eigen vectors of a linear transformation and matrices. Eigen space, Algebraic and Geometric Multiplicity of an eigenvalue. Diagonalisabilty of an n x n matrix over IR

Unit 4: (15 lectures)

Inner products spaces. Cauchy- Schwarz inequality, Triangle inequality, orthogonal vectors, orthogonal complement, orthogonal sets and bases, Gram-Schmidt Process

References:

1. Anton H, Elementary Linear Algebra, Wiley Pvt. Ltd.
2. Hoffman K. & Kunze R., Linear Algebra , PHI
3. Kumaresan S., Linear Algebra: A Geometric Approach, PHI
4. Strang G., Linear Algebra and its Applications, Cengage Learning
5. Vasishtha A.R., Linear Algebra, Krishna Prakashan

Course Title: ADVANCED ANALYSIS

Course Code: MAT-IV-E-5

Marks: 100

Credits: 4

Course Objectives: To develop the understanding of point wise and uniform convergence of sequence and series of real valued functions.

To understand Power Series and Fourier Series

Learning outcome: The student will be able to understand series and sequences of real valued functions

Prerequisites: Basic Real Analysis, Mathematical Analysis

Module I: Sequences of Real valued Functions (20 lectures)

Pointwise and uniform convergence and examples, M_n test for uniform convergence and examples, Cauchy's criterion for uniform convergence and examples.

Properties of uniformly convergent sequence of functions: uniform convergence and continuity, uniform convergence and differentiability, uniform convergence and integrability and examples

(i)Weierstrass Approximation Theorem

(ii)C [a, b] is complete, where C [a, b] is a class of continuous functions on C [a, b]

Module II: Series of Real Valued Functions (20 lectures)

Revision of series of real numbers, Alternating Series, Leibnitz test, rearrangement of terms of series, Pointwise and uniform convergence and examples, Cauchy's criterion for uniform convergence, Weierstrass M-test and examples

Properties: (i) uniform convergence and continuity, (ii) uniform convergence and differentiability, (iii) uniform convergence and integrability and examples

Module III: Power Series (20 lectures)

Definition of Power Series and examples. Uniform convergence of Power Series and examples, Theorems on Power series, radius and interval of convergence

Theorem: Power series can be integrated and differentiated in its interval of convergence, examples

Trigonometric functions (sine and cosine), exponential functions and logarithmic functions and their properties.Binomial Series

References:

1. Bartle R.G, Sherberf D.R, Introduction to Real Analysis (Third Edition), Wiley
2. Berberian S.K., A First Course in Real Analysis, Springer
3. Bhatia R., Fourier Series, Hindustan Book Agency
4. Goldberg R.R, Methods of Real Analysis, Oxford and lbh
5. Kumaresan S & Kumar A, A Basic Course in Real Analysis, CRC Press
6. Malik S.C. and Arora, Mathematical Analysis, New Age

7. Pugh C.C., Real Mathematical Analysis,Springer
8. Rudin W., Principles of Mathematical Analysis, Tata McGraw Hill
9. Shanti Narayan, P.K.Mittal, A Course of Mathematical Analysis, S Chand
10. Somasundaram, Mathematical Analysis
11. Stein E.M & Shakarchi R., Fourier Analysis: An Introduction, Princeton University Press

Course Title: ALGEBRA-II

Course Code: MAT-IV.E-6

Marks: 100

Credits: 4

Course Objectives: To introduce to students further concepts of groups and rings in Algebra

Learning outcome: The student will gain additional knowledge of groups and rings

Prerequisites: Basic Algebra, Algebra I

Unit 1

Review of basics of groups, Group actions, orbits and stabilizers, class equation. Cauchy's Theorem, Sylow Theorems, structure theorem for finite abelian groups. (Statement and examples)

Unit 2

Review of basics of rings, Ring of Polynomials, divisibility in a ring, irreducible and prime elements, Eucledian domains, Principle Ideal domains, unique factorisation domains.

Unit 3

Irreducibility of polynomials in one variable with integer/ rational coefficients, Gauss' lemma, Eisenstein's criterion for irreducibility, cyclotonic polynomials, examples

References:

1. M.Artin, Algebra, Pearsons
2. S. Lang, Undergraduate Algebra,
3. J. Stillwell, Elements of Algebra,
4. Fraleigh J.B., A First Course in Abstract Algebra
5. Gallian J, Contemporary Abstract Algebra, Narosa, New Delhi
6. Gopalkrishnan N.S., University Algebra
7. Herstein I.N , Topics in Algebra,(2012) John Wiley & Sons., 2ndEdition

Course Title: Operation Research

Course Code: MAT-IV.E-7

Marks: 100

Credits: 4

Course Objectives: This course aims to teach linear programming

Learning outcome: Students will be able to solve linear programing problems

1. Linear Programming Problem **(5 lectures)**

Definition of standard form, formulation of LPP, convex set and their properties, extreme points. Graphical solution of LPP (Only two variables).

2. Simplex Method: **(25 lectures)**

Theorems related to simplex method .and problems .Cases pertaining to existence of multiple solutions, unbounded and no feasible solution.Big M method and two phase Simplex method

3. Transportation Problems: **(8 lectures)**

Mathematical formulation, condition for existence of feasible solution, rank of transportation matrix, Initial basic feasible solution by (i) NWC method (ii) Matrix-minima and (iii) VAM, Modi's method to find an optimal solution, balanced and unbalanced transportation problems.

4. Assignment Problems: **(7 lectures)**

Mathematical formulation, Hungarian methods to solve assignment problems, balanced & unbalanced assignments problems

5. Game Theory **(15 lectures)**

Optimal Solution of Two-Person Zero-Sum Games, Solution of Mixed Strategy Games, Converting Game theory into LPP

References:

1. Kanti Swarup, Gupta P.K, Man Mohan, Operations research, S Chand
2. Loomba, Linear Programming
3. Taha H, Operation Research, Pearson
4. Vajda, Game Theory

Course Title: Cryptography

Course Code: MAT-IV E-8

Marks: 100

Credits: 4

Course prerequsites: Algebra-1 and Number Theory-1

Course objectives: - To introduce some applications of algebra and number theory to students.

Learning Outcome: -Students will learnsome methods which are used in data storage and transfer.

Unit 1: Review of some topics of Number theory and algebra :- **(12Lectures)**

Divisibility and primes, Prime factorization, Euclidian algorithm, Fermat's little theorem, Congruence and ring of integers.

Groups of compositions, order of permutation, Fields, finite fields, ring of polynomials, factorization of polynomials, to be reviewed.

Unit 2: Classical Cryptosystems. **(8Lectures)**

Affine cryptosystem, Hill cryptosystem, Block Ciphers, Stream Ciphers, Linear feedback shift registers.

Unit 3: Public Key cryptosystem. **(16Lectures)**

One way functions, Trapdoor functions, RSA Public Key cryptosystem, Key exchange protocols, hash functions.

Unit 4: Private Key cryptosystem. **(8Lectures)**

Modern techniques and algorithms like DSE and AES.

Unit 5: Elliptic curve cryptosystem. **(16Lectures)**

Introduction to elliptic curves and its application to factorization and cryptography.

References:

1. N. Koblitz, a course in Number theory and Cryptography, Springer.
2. J Katze & Y Lindell, Introduction to modern cryptography, Springer.
3. C Paar & J Pelze, Understanding Cryptography: A textbook for students and practitioners, Springer.
4. W Trappe, Introduction to Cryptography with coding theory, Pearson.

Course Title: Functions of several variables

Course Code: MAT-V C7

Marks: 100

Credits: 4

Course Pre Requisites: Mathematical Analysis, Co-ordinate geometry.

Aim: - To introduce multivariate calculus, i.e. Continuity, Differentiation and Integration of functions of several variables, applications of differentiation and integration.

Learning outcome: - Students will be well prepared to understand vector valued functions, can visualize lines, curves and surfaces in $\mathbf{R^3}$.

Unit 1 :- **(15 lectures)**

Introduction: -Neighbourhood of a point in $\mathbf{R^2}$(in$\mathbf{R^n}$), Open sets in $\mathbf{R^2}$, Sequence in $\mathbf{R^2}$, limit point of a sequence, Limit of a function, Continuity of a function. Functions from $\mathbf{R^2}$ to **R,**Examples, and graphs in three dimensions.

Unit 2:- **(15 lectures)**

Directional derivatives, Partial Derivatives, their geometrical meaning, MVT, higher order partial derivatives, Equality of mixed partial derivatives, Taylor's theorem, Applications of partial derivatives, Maxima-Minima, Saddle Point, Lagrange multiplier method,

Unit 3:- **(15 lectures)**

Differentiability of functions of two variables, sufficient condition for Differentiability, Total derivative, chain rule, Jacobian. Implicit and Inverse function theorems.

Unit 4:- **(15 lectures)**

Integration. Line integral, fundamental theorem of Calculus, Double integral on rectangles & bounded regions, change of order, change of variables, calculating volumes and surface areas of simple geometrical objects.

References:

- S.R. Ghorpade & B.V. Limaye, A Course in Multivariable Calculus and Analysis, Springer
- S.C. Malik & S. Arora, Mathematical Analysis, Fourth Edition, New Age International
- G.B. Thomas Jr. and R.L. Finney, Calculus and Analytic Geometry, Pearson
- W.H. Fleming, Functions of Several Variables, Springer
- T. Apostol, Calculus Volume II, Wiley India

Course Title: Metric Spaces

Course Code: MAT-V E-9

Marks: 100

Credits: 4

Course Objectives: To introduce different kind of 'Distance' and analysis that follows.

Learning outcome: Students will start to understand abstract nature analysis, also will help them to appreciate Euclidian analysis.

Prerequisites: Basic Real Analysis.

Unit1] Idea of 'Distance', definition, examples of Metric Spaces. Basic terminologies – neighbourhood of a point, open ball, open set, closed set, interior point, exterior point, limit point, isolated point, cluster point, closure of a set, interior of a set, frontier (boundary) of a set, Dense set, nowhere dense set, bounded set, diameter of a set, distance between the sets, distance of a point from a set and Results involving these concepts. **(15 lectures)**

Unit2] Equivalence of metrics, geometry with different metrics, Subspace of a metric space, results on subspaces of metric space, sequences in a metric space, convergence of sequences, Cauchy sequence, concepts of complete metric space, completion of a metric space. **(15 lectures)**

Unit3] Connected subsets of a metric space, Separation of a set, connected components of metric space, Results regarding connectedness of a metric space. **(10 lectures)**

Unit4] Compact metric space, open cover, finite cover, sequential compactness, one point compactification, **(10 lectures)**

Unit5] Functions on metric space. Continuous functions (maps), equivalent definitions, invariance of compactness, completeness, connectedness of continuous images. Open maps-closed maps and their basic properties. **(10 lectures)**

References:-

1. E.T. Copson, Metric Spaces, Cambridge University Press
2. G.F. Simmons, Introduction to Topology and Modern Analysis, Mc Graw Hill
3. S. Shirali and H.L. Vasudeva, Metric Spaces, Springer
4. S. Kumaresan, Topology of Metric Spaces, Narosa

Course Title: Differential Equations-II

Course Code: MAT-V E10

Marks: 100

Credits: 4

Course Objectives: Some more techniques of solving differential equations. Introduction to PDE.

Learning outcome: Students enhance their thinking and problem solving abilities.

Prerequisites: Ordinary Differential equations.

Unit 1: Power Series Solutions of DE

Series solution of first order DE, Series solution of second order Linear DE at ordinary points Regular singular points, Gauss Hyper geometric equation, Bessel's function Legendre Polynomials **(20 lectures)**

Unit 2: Laplace Transforms

Definition and examples, Derivatives and integrals of Laplace Transforms Applications to DE, Convolutions **(15 lectures)**

Unit 3: Systems of first order DE

Homogeneous Linear Systems with Constant coefficients **(10 lectures)**

Unit 4: Boundary value problems, Sturm-Liouville Theorems, linear differential equation of higher order. **(15 lectures)**

References:

- G.F. Simmons, Differential Equations with Application and Historical Notes, Mc Graw Hill.
- W.E. Boyce and R.C Di Prima, Elementary Differential Equations and Boundary value Problems, Ninth Edition, Wiley India
- M. Tenenbaum and H. Pollard, Ordinary Differential Equations, Dover

Course Title: Graph theory

Course Code: MAT-V E-11

Marks: 100

Credits: 4

Course Pre Requisites: Basic set theory, Principle of counting, Principles of Mathematical Induction

Course objective: To introduce the concept of discrete graphs, trees and some of their applications in real world problems.

Unit 1 Graphs (8 Lectures)

The Konigsberg 7 bridges problem and Euler's solution to it.

Definition of graph examples and types of graphs: complete, directed, bipartite, multi graph, etc, degree of a vertex, Adjacency matrix, incidence matrix, operations on graphs: subgraphs,

union, intersection and Cartesian product, isomorphisms in graphs, special graphs: self-complimentary graphs, Petersen graphs, Herschel's Graph

Unit 2 Walks paths and cycles (12 Lectures)

Definition of walks, paths and cycle, distance and weighted distance, radius and diameter of a graph, eccentricity of a vertex. Euler walks, Hamiltonian cycles application: travelling salesman problem

Unit 3 Connectivity, cut-points, blocks, connectivity(8 Lectures)

Unit 4 Trees (12 Lectures)

Definition of tree, results on trees, spanning tree, application: minimum spanning trees and Kruskal's algorithm. Breadth First Search Algorithm and its use in the Chinese postman problem. Dijkstra's Algorithm for shortest path.

Unit 5 Coloring (10 Lectures)

Vertex coloring, chromatic number of a graph and simple relation, chromatic polynomial and its properties, edge colorings.

Unit 6 Planarity (10 Lectures)

Representations and crossing number of graphs, Euler's formula, maps and planar graphs. Proof of five color theorem for planar graphs

References:

1. W.D. Wallis, A Beginners Guide to Graph Theory, Birkhauser
2. Robin J. Wilson Introduction to Graph Theory, Longman Group Ltd.
3. J.A. Bondy, U.S.R Murthy, Graph Theory, Springer
4. D.B. West, Introduction to Graph Theory, second edition, Prentice Hall of India.
5. F Harary, Graph Theory, Narosa
6. V.K. Balakrisnan, Graph Theory (Schaum series), Mc Graw Hill India

Course Title: PEDAGOGY OF MATHEMATICS

Course Code: MAT-V E-12

Marks: 100

Credits: 4

Course Objectives: To make the students aware of the different methods used for the teaching of mathematics.

Learning outcome: Students will be able to teach mathematics to school level children better.

Unit 1: Meaning and History of Mathematics **(10 lectures)**

Meaning of Mathematics

Branches of Mathematics

Historical Development of Mathematics

Unit 2: Methodology of Teaching Methods **(40 lectures)**

Inductive - Deductive

Analytic- Synthetic

Heuristic/ Discovery

Project Method

Open Ended Approach

Investigative Approach

Problem Solving

Concept Attainment Model

(With applications in different topics in School level Mathematics syllabus)

Unit 3: Techniques of Teaching **(8 lectures)**

Assignments

Drill work

Remedial teaching

Accelerated teaching

Unit 4: Qualities of a good Mathematics Teacher **(2 lectures)**

References:

1. E.T Bell, Men of Mathematics, Touchstone
2. M. Ediger, Essays on teaching mathematics, Discovery Publishing Pvt.Ltd
3. R.G. Goel, Teaching of Mathematics, Lotus Press
4. A. James., Methods of teaching Mathematics, Neelkamal
5. Kline Morris, Mathematical Thought From Ancient to Modern Times, Oxford University Press
6. S. Krantz, How to teach Mathematics, American Mathematical Society
7. G. Polya, How to Solve It, Penguin UK
8. Sidhu Kulbir Singh., Teaching of Mathematics, Sterling Publishers Pvt. Ltd
9. J. Stillwell, Mathematics and its History, Springer
10. P.P Zubair., Teaching of Mathematics, Aph Publishing

Course Title: Vector Analysis

Course Code: MAT-VI C-8

Marks: 100

Credits: 4

Course Objectives: To introduce students to Vector representations of geometrical objects, analysis done on them and their applications in Physics.

Learning outcome: Students are introduced to one more representation of geometrical objects and extensions of the fundamental theorem of integral calculus.

Prerequisites: functions of several variables.

Unit 1: Revision of vectors :- Basic concept such as dot product, cross product, scalar triple product, vector triple product, geometrical understanding of all algebraic operations, orthogonal vectors, vector equations of geometrical objects, **(10 lectures)**

Unit 2: Vector valued functions (vector fields), scalar valued functions (scalar fields), concepts of 'curves', 'plane', 'surface' in $\mathbf{R^3}$ Idea of continuous, smooth and regular objects in $\mathbf{R^3}$, Gradient, Divergence and Curl of these functions, Physical interpretations. Irrotational and solenoidal vector fields. **(20 lectures)**

Unit 3: Line integral, surface integral and volume integral, (arc length, surface area, and volume of simple objects), Green's theorem, Stokes theorem, Gauss Theorem and Green's formulas. **(20 lectures)**

Unit 4: Theory of Curves in $\mathbf{R^3}$, Unit speed curve, Tangent, Normal, Bi-normal, Curvature, Torsion, Ferret-Serrate formulae, evolutes and involutes. (**10 lectures)**

References:

1 .H.F. Davis and A.D. Snider, Introduction to Vector Analysis, Sixth Edition, Allyn& Bacon

2. J.E. Marsden and A. Tromba, Vector Calculus, Sixth Edition, W.H. Freeman Publishers

3. E. Kreysig, Advanced Engineering Mathematics, Tenth Edition, Wiley Inida

4. M. Spiegel, Vector Analysis: Schaum's Outline Series, Mc Graw Hill

Course Title: COMPLEX ANALYSIS

Course Code: MAT-VI E-13

Marks: 100

Credits: 4

Course Objectives: Introduce students to complex numbers, complex functions and complex differentiation and integration.

Learning outcome: The student will know about complex numbers and complex functions.

Prerequisites: Basic Real Analysis, Mathematical Analysis, Functions of several Variables

Unit 1: Complex Numbers

Algebraic properties of complex numbers, modulus, Argand diagram, exponential form and polar coordinates, triangle inequality and metric properties, connectedness of regions. **(10 lectures)**

Unit 2: Analytic Functions

Complex valued functions on complex domain, limits and continuity of complex valued function on a complex domain, differentiability and analytic functions, algebra of analytic functions, Cauchy-Riemann equations, sufficient condition for analyticity, Harmonic Functions. **(10 lectures)**

Unit 3: Elementary Functions

Exponential, logarithmic function and its branches, trigonometric functions, hyperbolic functions, complex exponents and roots. **(10 lectures)**

Unit 4: Contour Integration

Contours and contour integrals, Cauchy Goursat's theorem, Simply connected domains, Cauchy's integral formula, higher derivatives of analytic functions, Liouville's theorem, fundamental theorem of algebra, maximum modulus principle. **(10 lectures)**

Unit 5: Series

Convergence of series, Taylor series, Laurent series. **(10 lectures)**

Unit 6: Residue Theory

Singularities of a function, poles and essential singularities, residues at a singular point and its computation, Cauchy residue theorem **(10 lectures)**

References:

1. R.V. Churchill and J.W. Brown, Complex variables and Applications, Eight Edition, Mc Graw Hill
2. L. V. Ahlfors, Complex Analysis, Mc Graw Hill
3. A.R. Shastri, Complex Analysis, Laxmi Publications
4. M. Spiegel and S. Lipschutz, Complex Variable: Schaum's Outline Series, Mc Graw Hill
5. J.B. Conway, Functions of a Complex Variable, Narosa
6. S. Ponnusamy, Complex Analysis, Narosa

7. S. Ponnusamy and H. Silverman , Complex variables with Applications, Birkhauser
8. T.W. Gamelin, Complex Analysis, Springer
9. E.M. Stein and R. Shakarchi, Complex Analysis, Princeton Lectures in Analysis

CourseTitle: NUMBER THEORY-II

Course Code: MAT-VI E-14

Marks: 100

Credits: 4

Course Objectives: To learn about Primitive Roots, Quadratic reciprocity and Fibonacci numbers

Learning outcome:The student will gain knowledge about different concepts in number theory.

Prerequisites: Number Theory I

Unit 1: Primitive Roots and Indices: - **(15 lectures)**

Review of congruence, Chinese remainder theorem Euler phi theorem and Euler theorem. The order of an integer modulo n, Characterization of positive integers n for which primitive roots mod-n exist, The theory of Indices.(Structure of the group of units of Z/nZ as a product of cyclic groups.)

Unit 2: The Quadratic Reciprocity Law **(15 lectures)**

Euler's criterion, Legendre Symbol and its Properties, Quadratic Reciprocity, Quadratic Congruences with Composite moduli

Unit 3: Representations of numbers as sums of squares. **(15 lectures)**

Fermat's two squares theorem, Lagrange's four squares theorem, and statement of Warings problem.

Unit 4: Continued Fractions **(15 lectures)**

Finite Continued fractions, Infinite continued fractions, Dirichlet's theorem on approximation of a real number by a rational number, Liouville's theorem.

References:

1. D. Burton, Elementary Number Theory, Seventh Edition, Mc Graw Hill
2. Baker, A Concise Introduction to the Theory of Numbers, Cambridge University Press

3. Niven, H.S. Zuckerman and H.L. Montgomery, An Introduction to the Theory of Numbers, Fifth Edition, Wiley India
4. W.W. Adams and L.J. Goldstein, introduction to Number Theory, Prentice Hall
5. S.G. Telang and M.D. Nadkarni, Number Theory

Course Title: Theory of Probability

Course Code: MAT-VI E-15

Marks: 100

Credits: 4

Course Objectives: To understand how mathematical tools are applied to develop tools in Statistics and learn to use those tools.

Learning outcome: Application of Combinatorics

Prerequisites: Statistical methods.

Unit 1 Revision of Probability theory, Set theory. **(5 lectures)**

Unit 2: Probability as a function from super set of a non-empty set to the interval [0, 1]. Axioms of Probability.

Probability distributions, Discrete and continuous distributions, Probability Mass function, density function, Distribution function, Central tendencies, Expected values, Variance, Standard Deviation, Moments, Moment generating function, characteristic function, conditional expectations and distribution, random vectors and joint probability distributions, functions of random vectors, change of variable. **(20 lectures)**

Unit 3: Uniform, binomial, Poisons, Geometric, Hyper-geometric, Normal, Chi-square, beta, gamma, Students t, F, distributions, their distribution functions, graphs, statistic, relations. **(25 lectures)**

Unit 4: Modes of convergence, Weak and Strong laws of large numbers, Central limit theorem **(10 lectures)**

References:

1. P. Billingsley, Probability and measure, 2nd edition, John Willy &sons (SEA) Pvt. Ltd. 1995

2. P.G. Hoel, S.C. Port & C.J. Stone, Introduction to Probability, Universal Book Stall, New Delhi, 1998.

3. J.S. Rosenthal, A first look at Rigorous Probability Theory, World Scientific, 2000.

4. M. Woodroofe, Probability with applications, McGraw-Hill Kogakusha Ltd. Tokyo, 1975.

Course Title: Computers for Mathematics

Course Code: MAT-VI E-16

Marks: 100

Credits: 4

Course prerequsites: Basic knowledge of computers, a basic course in ODE, Numerical analysis. Familiarity with computer programming will be helpful but not compulsory.

Course objectives: - To train students to use computers for mathematical typing, computing and plotting. Helping them understand theory using computational methods.

Learning Outcome: -Students will be able to use these free packages for writing and drawing mathematical papers. Also can understand some basic aspects of programming.

Unit 1.Introduction to LaTeX (15 Lectures)

Mathematical typing in MS Word 07/13

Short comings of Word, need of some other package of writing.

Document Editing using LaTeX: - Text formatting, Paragraph formatting

Fonts and colours, special characters, Tables, Page layout, importing graphics, footnotes, referencing

Beamer Presentations

Unit 2.Mathematical typesetting in LaTeX (10Lectures)

Mathematical environments and packages

Symbols, Greek letters and operators, Powers and indices, fractions, Roots, Sums, integrals and derivatives, Brackets, Matrices

Unit 3. **Python** Some basics of programming, algorithms, flowcharts, syntax, do s and don't s, Python programming. **(15 lectures)**

Unit 4. Scilab/ Matlab (5Lectures)

Introduction to Scilab, familiarizing with the Scilab, Scilab syntax, Variables, functions, plotting graphs

Unit 5. Geogebra (3 lectures)

Unit 6. Numerical methods using Scilab/ Python (12Lectures)

Finding roots of algebraic and transcendental equations, Interpolation, Numerical integration and differentiation, fitting of data, Numerical ODE, Solutions of simultaneous equations

References:

1. M.Alley, the Craft of Scientific Presentations, Springer (2003).
2. W.C. Booth, G.G. Colomb, and J.M. Williams, The Craft of Research (Chicago Guides to Writing, Editing, and Publishing) Univ. of Chicago Press, 2008.
3. George Graetzer, Math into LaTex, An introduction to LaTex and AMS-LaTex, Birkhauser, 1996.
4. Donald E. Knuth; Tracy E Larrabee, Paul M. Roberts: Mathematical writing, Mathematical Association of America, 1989.
5. S. Krantz: A Primer of Mathematical Writing: Being a Disquisition on Having Your Ideas Recorded, Typeset, Published, Read & Appreciated, American Mathematical Society, 1996.
6. S. Krantz: How to Teach Mathematics, American Mathematical Society, 1999.
7. Leslie Lamport: LaTex, a document preparing system, Addison-Wesley, 1994.
8. Jr. Strunk, William; E. B. White, The Elements of Style, Fourth Edition, Longman; 4th edition (1999).

Course Title: COMPUTATIONAL LINEAR ALGEBRA

Course Code: MAT-IV E-17

Max Marks: 100

Credits: 4

Course Objectives: To learn about the different computational techniques in linear algebra

Learning Outcome: The student will be able to apply the techniques learnt to solve various problems in linear algebra.

Unit 1: Review of Gaussian Elimination (8 lectures)

Unit 2: Orthogonality (10 lectures)

Orthogonal vectors and subspaces, projections and least squares, QR Factorizations

Unit 3: Positive Definite Matrices (20lectures)

Maxima, minima and saddle points, test for positive definiteness, singular value decompositions, Lagrange multipliers and Rayleigh quotients, finite element method

Unit 4: Matrix norm and Condition Number (22 lectures)

References:

1. G. Strang, Linear Algebra and its Applications